

# Relaxation Schemes for Flows in Networks: Application to Shallow Water and Blood Flow Equations

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Modeling fluid dynamics in complex networks, such as water channels and arterial blood vessels, relies heavily on hyperbolic systems of conservation laws. A major challenge in simulating these systems is accurately handling the physical coupling conditions at network junctions. Traditionally, this requires complex approximate Riemann solvers, which can be computationally intensive and particularly restrictive when dealing with torrential or supercritical flow regimes. In this talk, a novel numerical scheme of relaxation-type is proposed to approximate such conservation laws on networks. Physical conditions at the junction are given and a novel strategy based on [1] is introduced to approximate the solution, avoiding the use of approximate Riemann solvers. This general approach allows for the explicit computation of solutions at the junction across both subcritical and supercritical cases [2]. Furthermore, we detail how the relaxation scheme is complemented with well-balanced reconstruction techniques to accurately treat physical source terms. Finally, we investigate the main properties of the numerical scheme and we validate the approach through several numerical simulations across different network configurations.

## References

- [1] Maya Briani, Roberto Natalini, and Magali Ribot. A relaxation scheme for the equations of isentropic gas dynamics on a network with jump transmission conditions. *accepted for publication in Commun. Math. Sci.*, 2026.
- [2] Tommaso Tenna. Relaxation schemes for flows in networks: Application to shallow water and blood flow equations. Preprint, arXiv:2512.02989 [math.NA] (2025), 2025.