BINNO: A First-Order Framework for Nonconvex and Nonsmooth Bilevel Optimization with Applications to Matrix Factorization

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Bilevel optimization with nonconvex and nonsmooth structures is both theoretically challenging and increasingly relevant in modern applications. In this talk, we propose BINNO, a novel first-order method based on proximal-gradient updates within an alternating minimization scheme, combined with descent principles from variational analysis.

The core idea is to couple the upper- and lower-level problems through a descent-driven averaging strategy, extending classical proximal methods to the bilevel nonconvex setting. We show that the method guarantees a descent property for a suitable surrogate of the bilevel objective.

We demonstrate the effectiveness of BINNO on sparse low-rank matrix factorization problems, combining ℓ_1 and nuclear norm regularization through a Frobenius data fidelity term. These models arise in several areas of numerical analysis, including inverse problems and signal processing, where our approach yields improved reconstruction accuracy compared to standard methods.

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