Discontinuous Galerkin methods for multiphysics models of neurodegeneration

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ABSTRACT

Neurodegenerative diseases impose a significant global burden, with many —including the Alzheimer's and Parkinson's diseases — classified as proteinopathies driven by the accumulation and spread of misfolded proteins exhibiting *prion-like* behavior. Mathematical modeling of prion dynamics plays a key role in understanding disease progression, and various frameworks have been developed to capture misfolding and aggregation processes at different levels of detail. In this work, we present and analyze a discontinuous Galerkin method on polytopal grids (PolyDG) for the semi-discrete approximation of nonlinear population dynamics models, with applications to the Fisher–Kolmogorov equation and the heterodimer model [1].

Neurodegeneration arises from tightly interconnected biophysical and biochemical mechanisms evolving across multiple spatial and temporal scales. Protein aggregation not only impairs neuronal function directly but also induces mechanical changes in brain tissue, contributing to progressive atrophy [2]. Disease progression is further shaped by the interplay between protein accumulation and vascular dynamics, particularly the relationship between amyloid-beta deposition and reduced cerebral blood flow [3]. To represent these coupled effects, multiphysics models have been developed that integrate the transport and accumulation of pathogenic agents with tissue deformation, degeneration, and vascular impairment. We demonstrate the effectiveness of this approach through high-order discontinuous Galerkin simulations in three-dimensional, patient-specific brain geometries reconstructed from magnetic resonance imaging.

REFERENCES

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